Robust Identification of Parametric Radiation Force Models via Impulse Response Fitting

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This paper presents a new identification method for obtaining parametric state-space models for radiation force computation. These state-space models can substitute the convolution integral in the equations of motion based on the impulse response function method. Thus, the method converts the integro-differential equation to an ordinary differential equation which reduces the computational effort of radiation force computation significantly. The identification is performed in time-domain which means that the retardation function is subject to fit. The method is verified by the application to a floating cylinder.

1 Introduction

In order to describe the dynamics of floating bodies in time-domain, state-of-the-art simulation methods typically use the impulse response function method also known as Cummins Equation [1]

\[ (M + A_{\infty})\ddot{\xi}(t) + \int_0^{\infty} K(\tau)\dot{\xi}(t-\tau)d\tau + S\xi(t) = f_{ex}(t), \]  

shown for zero forward speed here. In this equation of motion, \( \xi \) describes the perturbation of the floating body. \( M \) is the mass matrix of the dry rigid body and \( A_{\infty} \) the so-called added mass matrix due to the surrounding fluid. The retardation matrix \( K(\tau) \) accounts for fluid forces of radiating waves and their memory effects. \( S \) denotes the hydrostatic stiffness matrix and \( f_{ex} \) other external forces.

Due to the convolution terms in this method, the equations of motion are integro-differential equations, which are expensive to compute and unsuitable for general control purposes. In order to avoid the convolution terms, Cummins Equation can be substituted by a system of first order ordinary differential equations, a so-called state-space model.

\[ \mu = \int_0^{\infty} K(\tau)\dot{\xi}(t-\tau)d\tau \approx \dot{x} = Ax + B\dot{\xi} \]

Several identification methods have been developed to obtain the parameters which constitute the state-space matrices \( A, B, C \) that represent the same dynamics as the Cummins Equation [3–5]. Nevertheless, their entire application is limited to a few academic cases of low order state-space systems which is mostly due to the lacking robustness of the identification methods.

2 Robust impulse response fitting

In this study we concentrate on time-domain identification methods, which means that the retardation matrix \( K(\tau) \) is subject to fit. The parameter identification problem is then described by the non-linear optimization problem

\[ \min \int_{\Omega} \| \hat{K}(\tau) - K(\tau) \|_F d\tau \]  

subject to

\[ x\bar{D}(\omega)x^T \geq 0 \]  

\[ xAx^T < 0 \]

where the objective function is defined as the Frobenius norm of the deviation of the fitted retardation matrix which is typically regarded as the overall fitting error. The fitted retardation matrix \( \hat{K}(\tau) \) is computed from the state-space matrices \( A, B, C \) including the fitting parameters by

\[ \hat{K}(\tau) = C \exp(A\tau)B. \]
Constraint (4) requires the obtained state-space system to be passive, which is the case if the fitted added damping matrix

$$\tilde{D}(\omega) = \Re \{ C[i\omega I - A]^{-1} B \}$$

is positive semi-definite for all $\omega \geq 0$. Here, $I$ denotes the identity matrix and $i$ the imaginary unit. The second constraint (5) requires the obtained state-space system to be stable which is the case if the state-space system matrix $A$ is negative definite. Both constraints are absolutely necessary to use the obtained state-space model in a time-domain simulation. Otherwise it is much likely that the simulated motions build up and yield completely wrong motions.

The non-linear optimization problem (3) with the non-linear constraints is ill-posed and generally not solvable with standard optimization algorithms. Therefore, we apply the following tailor-made two-step fitting procedure. In a first step, an optimal solution to the unconstrained optimization problem is found by Prony’s method [6]. Thereafter, this solution is slightly modified in a second step so that the constraints (4) and (5) are fulfilled [2]. Thus the method works robustly even if high fitting orders or multiple degrees of freedom make the stability and passivity constraints difficult to fulfil.

3 Application

For verification purpose, the method is applied to simulate the dynamics of a floating cylinder. Table 1 and figure 1 describe the test case which was taken from the literature, [4].

**Table 1: Test case specification.**

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<tbody>
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<td>radius</td>
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<tr>
<td>draft</td>
<td>$c$</td>
</tr>
<tr>
<td>mass</td>
<td>$m$</td>
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<tr>
<td>water depth</td>
<td>$h$</td>
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<tr>
<td>water density</td>
<td>$\rho$</td>
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<tr>
<td></td>
<td>$1000 \text{ kg/m}^3$</td>
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As shown in figure 2, the presented method provides a very good fit of the retardation function of the heave motion of the cylinder. The fitting quality increases with the fitting order so that a fit of order 6 is not distinguishable from the original retardation function in figure 2. In a time domain simulation (figure 3), the heave motion and the damping effect solely due to radiation forces is also in good to very good agreement with the results published in [4].

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**References**


