ABSTRACT

The reliable and correct determination of the wave resistance is the key to an efficient ship design, as the wave resistance is one of the major factors concerning building and operational costs. In this paper the authors present a new approach to transverse wave cut analysis to overcome the major shortcoming of the resistance prediction from potential flow: a numerically stable and physically sound estimate for the magnitude of the wave making resistance.

INTRODUCTION

The calm water resistance determination is of significant importance during the early phase of the ship design process, as it affects a large number of design parameters. The most important of these parameters is the required break power of the main engine and subsequently the fuel storage capacity, as well as an important part of the operational cost. From this fact one can conclude that the calm water resistance is one of the prime efficiency indicators of a particular design.

The calm water resistance is made up in the majority by two resistance components: the frictional resistance and the wave making resistance. For the determination of the resistance three widely independent procedures are available. Firstly; towing tank tests, which also act as obligatory reference for the sea trials and for the fulfillment of the contract specifications. Secondly; empirical formulae, derived from towing tank test and analytical or semi-analytical considerations. Numerical methods form the third group of procedures. These methods can roughly be divided into two groups: finite volume based methods, that typically solve the RANS equations, and boundary elements methods that solve Laplace’s equation. From these two groups the latter are best suited for the early design, due to their short response time. Although they lack the ability to capture the frictional resistance, they can give very good impressions of the secondary wave system, who’s minimization is the main objective in wave resistance optimization.

Although pressure integration is available to determine the wave making resistance from potential flow, the reliability of those numbers is considered poor. Therefore an alternative approach, which is independent from the pressure integration and the body discretization, is called for. Such an approach is given by the so called wave cut analysis. The wave cut analysis is based on energy balance considerations. Energy flows into the system through the moving ship and is transported downstream by the secondary wave system. The wave making resistance can be interpreted as this energy flow divided by the system velocity. In this paper the authors present two new approaches to the so called transverse wave cut analysis method, which are able to give reliable and accurate predictions of the wave making resistance. In the following sections the authors give an short overview over the underlying theory of the calm water resistance, before they discuss the underlying theory of the transverse wave cut analy-
sis as it is found in literature toady. Next the new approaches is presented before the numerical scheme of the improved wave making resistance prediction is outlined. Finally test cases and validation results are presented, before the paper closes with a short summary, conclusion and outlook.

THEORETICAL BACKGROUND

In this section the theoretical background of the calm water resistance determination is shortly presented.

CALM WATER RESISTANCE

The assessment of the calm water resistance follows Froude’s hypothesis, which states that the total resistance can be divided into two parts and consequently be determined by model testing. These two parts are the wave making resistance dominated by gravitation and the frictional resistance dominated by viscosity, usually connected with Froude number $F_N$ and Reynolds number $R_E$.

$$R_T = R_F(R_E) + R_W(F_N). \quad (1)$$

Model tests have to be performed with models fulfilling the laws of similarity, which are in particular the geometrical similarity and temporal similarity. While the geometrical similarity is easily handled the temporal similarity is not. The scaling factor for time is deduced either from Froude’s number or Reynold’s number leading to $t_m = \frac{1}{\sqrt{\lambda}} \cdot t_S$ in the first and to $t_m = \frac{1}{\lambda^2} \cdot t_S$ in the second case. For any scale other then $\lambda = 1$ both cannot be fulfilled at the same time.

Although towing tank tests according to Froude demand the wave resistance to be determined, there is no simple way to do so and the typical procedures runs the different way round: the viscous resistance is determined and subtracted from the measured total resistance.

$$c_R = c_T - c_{F0} \quad (2)$$

The remaining part – the residuary resistance – is then treated as if it were the wave resistance. For the determination of the frictional resistance coefficient $c_{F0}$ today usually the ITTC57 correlation line is used, which was proposed in 1957 by the International Towing Tank Conference (ITTC) as a replacement of another friction line and was regarded as an “interim solution” (p. 324, [1]). The ITTC57 correlation line reads:

$$c_{F0} = \frac{0.075}{(log_{10}(Re) - 2)^2}. \quad (3)$$

The ITTC57 correlation line is intended to cover the influence of the body, so implicitly adding a form factor to the flat plate friction line. To overcome limitations caused by significant deviations of this implicit form factor the ITTC78 [2] procedure recommends to take an additional form factor according to Eqn. 4 into account. The application of this procedure in towing tank test is not mandatory and therefore one often finds $(1 + k) = 1$.

$$c_R = c_t - (1 + k)R_{F0} \quad (4)$$

The recommended method for the form factor determination [2] is Prohaska’s method as explained below: The values for $\frac{c_T}{c_{F0}}$ are plotted according to Eqn. 5 and then a straight line fit of the sample is extrapolated to $\frac{c_T}{c_{F0}} = 0$ (cf. Fig. 1). The form factor $(1 + k)$ is the the ordinate of the extrapolated function.

$$\frac{c_T}{c_{F0}} = (1 + k) + k \frac{F_N}{c_{F0}} \quad (5)$$

Solvers for the Navier-Stokes equations may typically integrate the pressure and shear stress over the surface:

$$R_p = \int_S p n_x \, dS, \quad (6)$$

$$R_\tau = \int_S \tau_x \, dS \quad (7)$$

and attribute them to $R_R$ and $R_{F0}$ accordingly. Potential flow solvers can only evaluate the pressure integral over the surface as they lack any viscosity related effects, the frictional resistance is typically determined according to the ITTC57 correlation line.
**TRANSVERSE WAVE CUT ANALYSIS**

An alternate approach for the determination of the wave resistance is available through the so called wave cut analysis. This paper limits itself to the so called transverse wave cut analysis in opposition to other methods namely the so called longitudinal wave cut analysis. More information about the latter and a detailed literature overview can be found in [4]. In general wave cut analysis methods were developed for the towing tank to enable a straightforward application of Froude’s hypothesis. In the current work the transverse wave cut analysis is solely employed for potential flow, although the authors hope that their findings might be usefully applied in experimental fluid dynamics.

The derivation of the wave resistance from the transverse wave cut analysis as it is found in the literature can be achieved in two different ways, which will both be presented in the following section as they play a key role for the current work. The first approach uses momentum continuity in the flow domain and the second the total energy flux over the domain borders. Both derivations were published in the first place by Eggers in 1961 in [5] and [6]. The aim of Egger’s procedure is to find the wave resistance with the surface deformation, i.e. the wave pattern, which can be measured during towing tank experiments. The measured wave signal is transformed into a Fourier series alone, which can be measured during towing tank experiments.

Equation 8 transforms into Eqn. 9 by replacing the temporal derivative with Euler’s equation and employing Gauss’ theorem to transform the volume integral into a surface integral, which is computed for each surface S of the flow domain individually (cf. Fig. 2).

\[
\frac{dQ_i}{dt} = \int_{\text{SB, PS, IN, OUT, B, H, FS}} \left[ -p n_i + \rho u_i (u_i n_x - u_k n_k) \right] dS + \int_{\text{V}} \rho f_i dV
\]

(9)

Leaving out vanishing terms, given by no flow through material boundaries \(p u_i (u_i n_x - u_k n_k) = 0\) (FS, H, and B), atmospheric pressure at the free surface \(p_n = 0\) (FS) and a steady state condition \(\frac{dQ_i}{dt}\) together with the replacement of \(\int -p n_i dS = F_i\) yields:

\[
F_i = \int_{\text{SB, FS, IN, OUT, B}} p n_i dS - \int_{\text{SB, PS, IN, OUT}} \rho u_i (u_i n_x - u_k n_k) dS - \int_{\text{V}} \rho f_i dV
\]

(10)

Since only the x-component has to be considered, faces SB, PS, B as well as the volume forces do not contribute:

\[
F_x = \int_{\text{IN}} p dS - \int_{\text{OUT}} p + \rho u (u_i n_x - u_k n_k) dS
\]

(11)

By replacing the pressure with Bernoullie’s equation,

\[
p = -\rho g z + \frac{1}{2} \rho (2u_\infty u - u^2 - v^2 - w^2)
\]

(12)

rearranging the terms and partly executing the integration \(\int dz\) one finally gets following expression for the wave resistance \(R_w = F_x\):

\[
R_w = \frac{1}{2} \rho g \int_{-B/2}^{B/2} \int_{-h}^{h/2} \left( -u^2 + v^2 + w^2 \right) dz dy,
\]

(13)

where \(-B/2\) and \(B/2\) denote the left and right domain border, \(h\) the bottom of the domain and \(\zeta\) the surface elevation. Eqn. 13 is linearized with respect to \(\zeta\) by setting

\[
\zeta = \frac{1}{g} u_\infty u(x,y, \zeta = 0)
\]

(14)

in the first term and \(\zeta = 0\) for the boundary in the second term. The velocity components are replaced by the gradient of a velocity potential function:

\[
\vec{u} = (u, v, w)^T = \nabla \Phi = (\Phi_x, \Phi_y, \Phi_z)^T
\]

(15)

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1 [6] is basically the English transcript of [5], which was published in German.
Inserting Eqn. 17 into Eqn. 13 yields:

\[ R_W = \frac{1}{2} \rho \int_{-b/2}^{B/2} \frac{1}{8} u^2 \nabla^2 \Phi^2 dy + \frac{1}{2} \rho \int_{-\beta/2}^{\beta/2} \int_{-\beta/2}^{\beta/2} (-\Phi_x^2 + \Phi_y^2 + \Phi_z^2) dz dy \]

The velocity potential is given by the potential of superimposed free wave components with different wave numbers \( k_n \).

\[ \Phi(x,y,z) = \frac{g}{u_{\infty}} \sum_n (a_n \cos(k_n x) - b_n \sin(k_n x)) \frac{\cosh(k_n(z-h)) \cos(k_n y)}{k_n \cosh(k_n h)} \]

Inserting Eqn. 17 into Eqn. 13 yields:

\[ R_W = \frac{1}{2} \rho g B \sum_{n=\infty}^{\infty} (a_n^2 + b_n^2) \left[ 1 - \frac{g \tanh(k_n h)}{2u^2_k k_n} \left( 1 + \frac{2k_n h}{\sinh(2k_n h)} \right) \right] \]

The second approach follows from energy considerations: a portion of the effective power given by \( P = R_W u_{\infty} \), flows via the secondary wave system into the flow domain, as depicted in Fig. 3. The forward boundary and the boundaries abreast of the body are considered so far away that no energy flows through them. Therefor all energy leaves the domain over the downstream boundary. The energy flow of a regular wave leaves the domain then with a velocity of

\[ u = u_{\infty} - c_{gr} \cos(\alpha), \]

where \( u_{\infty} \) is the system velocity, \( c_{gr} \) the group velocity of the regular wave and \( \alpha \) the propagation angle. The energy flow of a regular wave is then the specific wave energy \( \bar{\epsilon} \) integrated over the width of the domain:

\[ \bar{\epsilon} = R_W \cdot u_{\infty} = \int_{-b/2}^{b/2} \tau dy \cdot (u_{\infty} - c_{gr} \cos(\alpha)). \]

Using the dispersion relation for shallow water

\[ c = \sqrt{\frac{g}{k} \tanh(k h)} = u_{\infty} \cos(\alpha), \]

the group velocity

\[ c_{gr} = \frac{c}{2} \left( 1 + \frac{2kh}{\sinh(2kh)} \right) \]

and dividing by the system velocity \( u_{\infty} \) Eqn. 20 can be rewritten as:

\[ R_W = \int_{-\beta/2}^{\beta/2} \tau dy \left[ 1 - \frac{g \tanh(k_n h)}{2u^2_k k_n} \left( 1 + \frac{2k_n h}{\sinh(2k_n h)} \right) \right] \]

For linearized wave theory the specific energy per horizontal area is given by:

\[ \bar{\epsilon} = \frac{1}{2} \rho g |\xi|^2. \]

The wave height \( \xi \) is approximated by the superposition of \( N \) free wave components:

\[ \xi(x,y) = \sum_{n=N/2}^{N/2} a_n \cos(\beta_n) + b_n \sin(\beta_n) \]

with

\[ \beta_n = k_n (\cos(\alpha_n) \cdot x + \sin(\alpha_n) \cdot y) \]

Replacing the specific energy in Eqn. 23 finally yields:

\[ R_W = \frac{1}{2} \rho g B \sum_{n=\infty}^{\infty} (a_n^2 + b_n^2) \left[ 1 - \frac{g \tanh(k_n h)}{2u^2_k k_n} \left( 1 + \frac{2k_n h}{\sinh(2k_n h)} \right) \right] \]

In Eggers procedure the Fourier coefficients are found from two consecutive wave cuts for different wave numbers \( k_n \) and propagation angles \( \alpha_n \).

**CURRENT WORK**

For the current work two alternate approaches are taken which resemble the above described derivations from momentum and energy considerations.

**MOMENTUM APPROACH**

For the direct momentum approach the wave elevation \( \xi \) in the first integral and the velocity components \( u, v \) and \( w \) in the second integral of Eqn. 13 are replaced by the flow potential \( \Phi \). The flow potential is considered to be superimposed from a
uniform potential $\Phi_e$ and a disturbance potential $\Phi_D$. Without any further limitations we replace

$$\zeta = \frac{1}{2g} |\nabla \Phi_D|^2$$  \hfill (28)

and

$$\vec{u} = (u, v, w)^T = \nabla \Phi = (\Phi_x, \Phi_y, \Phi_z)^T,$$  \hfill (29)

which yields:

$$R_w = \frac{1}{2} \rho \left[ \frac{B/2}{4} (\nabla \Phi_D)^2 dy + \frac{B/2}{h} \int_{-B/2}^{B/2} (\Phi_x^2 + \Phi_y^2 + \Phi_z^2) dz dy \right]$$  \hfill (30)

The integrals in Eqn. 30 can be evaluated numerically by providing sufficient evaluation points for $\nabla \Phi$ in a transverse plane sufficiently far behind the disturbance.

**ENERGY APPROACH**

The specific wave energy, that is the energy per horizontal area, needed by Eqn. 23 consists of two parts, the potential energy and the kinetic energy. The potential energy is given by the lifting height times the weight per area:

$$e_{pot} = -\frac{\zeta}{2} \rho g (-\zeta) = \frac{1}{2} \rho g \zeta^2.$$  \hfill (31)

Averaging Eqn. 31 over one wavelength $\lambda$ yields:

$$\bar{e}_{pot} = \frac{1}{\lambda} \int_0^\lambda \frac{1}{2} \rho g |\zeta|^2 \cdot \sin^2 \left( \frac{2\pi}{\lambda} x \right) dx = \frac{1}{4} \rho g |\zeta|^2.$$  \hfill (32)

The specific kinetic energy is given by the integral of the orbital velocities over the water depth:

$$e_{kin} = \int_\zeta \frac{1}{2} \rho \omega^2 |\zeta|^2 \exp(-2kz) dz = \frac{1}{4} \rho g |\zeta|^2 \exp(-2k|\zeta| \sin(2\pi/\lambda \cdot x)).$$  \hfill (33)

Averaging again over one wavelength $\lambda$ yields:

$$\bar{e}_{kin} = \frac{1}{\lambda} \int_0^\lambda \frac{1}{4} \rho g |\zeta|^2 \exp(-2k|\zeta| \sin(2\pi/\lambda \cdot x)) dx.$$  \hfill (34)

Both together form the total mean energy per horizontal area:

$$\bar{e} = \frac{1}{4} \rho g |\zeta|^2 \left( 1 + \frac{1}{\lambda} \int_0^\lambda \exp(-2k|\zeta| \sin(2\pi/\lambda \cdot x)) dx \right)$$  \hfill (35)

The final steps follow the procedure as already described above. The wave height $\zeta$ is replaced by Eqn. 25 and Eqn. 35 subsequently inserted in Eqn. 23. Introducing the abbreviations

$$\tilde{a}_n = a_n \cdot \left( 1 + \frac{1}{\lambda} \int_0^\lambda \exp(-2k_n a_n \cos(2\pi/\lambda_n \cdot x)) dx \right)$$  \hfill (36)

and

$$\tilde{b}_n = b_n \cdot \left( 1 + \frac{1}{\lambda} \int_0^\lambda \exp(-2k_n b_n \cos(2\pi/\lambda_n \cdot x)) dx \right)$$  \hfill (37)

finally yields an expression very similar to Eqn. 27:

$$R_w = \frac{1}{4} \rho g B \sum_{n=-\infty}^{\infty} (\tilde{a}_n^2 + \tilde{b}_n^2) \left[ 1 - \frac{g \tanh(k_n h)}{2u_n k_n} \left( 1 + \frac{2k_n h}{\sinh(2k_n h)} \right) \right]$$  \hfill (38)

The integral $\exp(sin(x))$ has no analytical solution and has to be treated numerically. For $\tilde{a}_n = 1$ and $\tilde{b}_n = 1$ Eqn. 38 transforms into the linearized expression given by Eqn. 27.

**NUMERICAL SCHEME**

Both of the above described methods where implemented as supplementary method for the potential flow solvers within the design frame work $E4$ (cf. [7]). The implementation according to Eqn. 30 is referred to as: Transversal Wave Cut Analysis from Potential flow (TWACP) and the implementation according to Eqn. 38 is referred to as Transversal Wave Cut Analysis from Fourier coefficients (TWACF). Additionally the linearized formulation according to Eqn. 27 is implemented and is referred to as: Linearized Transversal Wave Cut Analysis from Fourier coefficients (LTWACP).

In a first step a conventional potential flow solver is used to determine the flow field and wave pattern of a vessel of interest. For this paper the solver MINK [8], which is currently under development at the Institute for Ship Design and Ship Safety of
the Hamburg University of Technology was used. MINK solves Laplace’s equation for the wave making problem by distributing Rankine sources over the body and free surface discretization. The non linear free surface boundary condition is solved iteratively with a Newton scheme; the computation can be performed with fixed or free trim. A typical computation with grid sizes up to 500 boundary elements takes about 7-15 iterations and a time to solution of less then 10 minutes.

The only required input from the solver for the wave cut analysis is the potential $\Phi$ as text file, i.e. the source strengths $q_i$ and the source locations $\vec{x}_i$, together with a few information about the computational domain, that are taken from the case input file of the potential flow solver. The required user input is kept to a minimum: evaluation scheme (TW ACP, TW ACF or LTWACP), number of wave cuts, number of points for the reconstruction of the surface in each wave cut, the wetted surface of the vessel for the determination of the wave resistance coefficient and additionally for the Fourier based schemes the number of Fourier coefficients to be determined.

For the TWACP Eqn. 30 is solved numerically by providing sufficient evaluation points for $\nabla \Phi$. The Potential $\Phi$ and the corresponding velocity filed are given by Eqn. 39 and Eqn. 40:

$$\Phi(\vec{x}) = \Phi_w + \Phi_D = \vec{u}_w \cdot \vec{x} - \sum_{i=1}^{n} \frac{q_i}{4\pi} \frac{1}{|\vec{x} - \vec{x}_i|}$$ (39)

$$\nabla \Phi(\vec{x}) = \vec{u}_w + \nabla \Phi_D = \vec{u}_w + \sum_{i=1}^{n} \frac{q_i}{4\pi} \frac{1}{|\vec{x} - \vec{x}_i|^3} (\vec{x} - \vec{x}_i)$$ (40)

The number of points for the numerical integration $\int dy$ is given by the user, while the integral $\int dz$ is solved with an initial step size of $dz = 1cm$ and terminates at a water depth of vanishing significant orbital velocities.

For the TWACF the water surface at the transverse cut positions is reconstructed from the potential $\Phi$ according to Eqn. 41.

$$\zeta = \frac{1}{2g} (\nabla \Phi_D)^2$$ (41)

By reconstructing the surface elevation the procedure becomes independent from the original surface discretization and eventually expensive intersection algorithms for non fitting coordinates. The typical number for the reconstruction of one transversal cross section of the free surface lies between 100 and 500 points ensuring a sufficiently smooth curve. The number of cross sections lies between 2 and 20 (50), though 10 is a good default number. The program runs in a loop so that it can be rerun with an adjusted set up, if the previous set up appears to be not satisfactory. In fact it is recommended to run several set ups till convergence is achieved. The Fourier coefficients are determined from an overdetermined equation system including all evaluated points by a least square approach. The number of Fourier coefficients is part of the user input.

While the TWACP returns wave resistance values for each cut independently, the TWACF returns the average of all cuts. The TWACP approach is about ten times slower due to the additional computation for the integral $\int dz$. Still the overall time consumption for a typical set up is only a few minutes. Although the TWACF is more time consuming the procedure has its virtues: as it delivers individual numbers for each cut the output can be used as a quality indicator of the original free surface discretization from the solver. From the underlying theory the resistance values for each cut must be equal. From larger deviations of these values one can conclude that the free surface has not been sufficiently discretized and that essential features of the wave pattern, and hence the resistance, might not be correctly captured.

RESULTS

For the evaluation of the above described theoretical findings and their numerical implementation two test cases where prepared. For a general validation the well known Wigley test case was chosen. For a real world scenario a modern design of a fast RoRo Ferry was chosen. The respective findings are presented below.

WIGLEY HULL

The first reference case is the well known Wigley hull. The hull form was especially developed by Wigley in the 1930’s for the experimental and analytical assessment of the wave resistance problem [9]. The hull form comprises parabolic sections below the waterline and straight sections above the waterline. Towards the ships ends the sections become narrower and form straight lines for the stem and stern. Today’s analytical form description reads

$$\frac{2y}{B} = \left(1 - \frac{z^2}{D^2}\right) \left(1 - \frac{4x^2}{L^2}\right) ; \text{ for } z \in [0...D]$$ (42)

below the waterline and

$$\frac{2y}{B} = \left(1 - \frac{4x^2}{L^2}\right) ; \text{ for } z \in [-\infty...0]$$ (43)

above the waterline, where $L$ is the length, $B$ the beam and $D$ the draft of the body. The origin of the coordinates is located at the
height of the still water line, midships in x- and y-direction. The x-axis shows forward, y to starboard and z downward. The most commonly used Wigley hull form, that is also used in this paper, is defined by the following main ratios: \( L / B = 10 \) and \( L / D = 16 \). The block coefficient results to \( C_B = 0.444 \).

**FIGURE 4.** COMPARISON OF THE WAVE RESISTANCE COEFFICIENTS FOR THE FIXED MODEL. COMPARISON INCLUDES EFD, PRESSURE INTEGRATION (PRESSURE), LINEARIZED TRANSVERSE WAVE CUT ANALYSIS (LTWCAF) AND NON LINEAR TRANSVERSE WAVE CUT ANALYSIS (TWCAF) AND (TWCAP)

For this Wigley hull exists a large amount of experimental and computational reference data. Experimental reference data (EFD) included in this paper is taken from [10–13]; computational reference data (CFD) is taken from [11, 13–18]. A summary of the data can also be found in [4]. The potential \( \Phi \) required by wave cut analysis is computed with the potential flow solver MINK, which also provides pressure integration for comparison (PRESSURE).

For the Wigley hull four different resistance curves are produced: pressure integration from potential flow (PRESSURE), linearized wave cut analysis from Fourier coefficients (LTWCAF), non linear transverse wave cut analysis from potential (TWCAF) and non linear transverse wave cut analysis from Fourier coefficients (TWCAF). All computations were performed for a free running and a fixed model. The computed velocity range covers \( F_N = [0.15..0.55] \).

Figures 4 and 5 show the results for the fixed running model in comparison with the experimental reference data (EFD) and the computational reference data (CFD) respectively. For the low and medium high velocity range \( (R_N = [0.15..0.4]) \) we find a close agreement of the pressure integration, TWACF and TWACP. Anyhow the TWACP tends to show the resistance for the hollow and hump Froude numbers accentuated. In the higher velocity ranges \( F_N = [0.4..0.55] \) we find a marked discrepancy between the pressure integration and non linear wave cut analysis on the one hand and between the TWACP and TWACF on the other hand. The highest values are reached with TWACP. Remarkably the values for both wave cut analysis coincide almost exactly for \( F_N = 0.55 \). Opposed to that one finds a substantial difference between LTWCAF and the three other curves over the whole velocity range. The curve reaches only about 70% of the values obtained from pressure integration. Compared with the available experimental (EFD) and numerical (CFD) data we find good agreement for TWACP and TWACF. The shortness of the results obtained from the LTWACP indicates that the effort of deriving full non linear expressions is justified.

Figures 6 and 7 show the results for the free running model compared to the available experimental reference data (EFD) and the available computational reference data (CFD) respectively.

In general these results repeat the findings for the fixed model, only for the higher velocity range there is an increasing deviation from the experimental reference data (EFD). A possible explanation might arise from considering Fig. 8, which shows the dynamic trim angle and the dimensionless sinkage of the free running model, compared to experimental data. One observes a general phase shift of the sinkage and a marked deviation of the trim angle beyond \( F_N = 0.35 \). The origins for this marked discrepancy has to be investigated. One plausible source might very well be flaws in the solver MINK, but this was neither established nor finally resolved before the paper’s dead line.
FIGURE 6. COMPARISON OF THE WAVE RESISTANCE COEFFICIENTS FOR THE FREE RUNNING MODEL. COMPARISON INCLUDES EFD, PRESSURE INTEGRATION (PRESSURE), LINEARIZED TRANSVERSE WAVE CUT ANALYSIS BY FOURIER COEFFICIENTS (LTWCAF) AND NON LINEAR TRANSVERSE WAVE CUT ANALYSIS BY FOURIER COEFFICIENTS (TWCAF) AND (TWCAP).

RORO FERRY

The second reference case is a fast RoRo ferry with the main particulars given in Tab. 1. This design is chosen as it shows only a moderate wave resistance. The reference data is taken from the original towing tank report [19]. The model test was performed free running covering a velocity range of \( Fn = [0.209...0.279] \). No form factor is included for the correction of the reported viscous resistance. The potential \( \Phi \) required by wave cut analysis is computed with the potential flow solver MINK, which also provides pressure integration for comparison (PRESSURE).

TABLE 1. MAIN PARTICULARS OF REFERENCE VESSEL

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, over all</td>
<td>199.8 m</td>
</tr>
<tr>
<td>Length, betw. perp.</td>
<td>190.3 m</td>
</tr>
<tr>
<td>Breadth, moulded</td>
<td>26.5 m</td>
</tr>
<tr>
<td>Draft, design</td>
<td>6.95 m</td>
</tr>
<tr>
<td>Velocity, design</td>
<td>22.5 kn</td>
</tr>
<tr>
<td>Froude Number</td>
<td>0.261</td>
</tr>
</tbody>
</table>

Figure 9 shows the curve of the residuary resistance coefficient for the full scale extrapolation (EFD) together with the corresponding curves from pressure integration from potential flow (PRESSURE), linearized wave cut analysis from Fourier coefficients (LTWCAF), non linear transverse wave cut analysis from potential flow (TWCAP) and non linear transverse wave cut analysis from Fourier coefficients. TWACP and TWACF show almost equal results, while LTWACF shows an increasing deviation from these two curves with increasing Froude number. The pressure integration (PRESSURE) shows a somewhat irregular behavior. For the first four sample points the resistance value is almost constant. The next value appears to be an outlier compared to the last three points of the curve, that shows the same.
slopes as the TWACP and TWACF at a somewhat lower level. This behavior resembles the problem with pressure integration from potential flow solvers: minor changes of the set up might lead to unsteadiness in the results. Anyhow, none of the curves appear to resemble the reverence data. As reason for this discrepancy the disregard of an additional form factor can be identified. For the form factor determination the available data from the towing tank report is arranged according to Prohaska’s method. Therefore \( \frac{c_T}{c_F_0} \) is plotted over \( F_n^{1/4} c_F_0 \) (cf. Fig. 10). In contrast to Prohaska’s method a linear instead of a constant form factor is assumed: therefore two form factors are generated for the lower half of the sample (k1) and for the upper half of the sample (k2). For these groups best fit straight lines are generated and extrapolated to \( F_n^{1/4} c_F_0 = 0 \). The values read: \( k1 = 1.155 \) and \( k2 = 1.103 \).

Figure shows original reverence curve, curves with both constant form factors applied and a curve for a linear interpolated form factor together with the results from TWCAF. The resulting curve with a linear form factor applied shows now very good agreement with the wave cut analysis.

CONCLUSION AND OUTLOOK

This paper presents two new ways of determining the wave making resistance from potential flow by using transverse wave cut analysis. The basis for the presented work is the derivation of the transverse wave cut analysis initially given by Eggers in 1962. The first approach uses simply the resulting equation from momentum considerations and evaluates the required wave elevation and orbital velocities from the flow potential returned by an potential flow solver. The second approach follows the original idea further, but starts from energy considerations. Instead of using the linearized wave energy a fully non linear derivation especially for the kinematic energy component is taken. This in particular is new, since basically all publications that are available to the authors use Eggers or almost similar approaches, typically ending with a linearized formulation. Both methods are able to deliver very good results concerning the presented test cases and constitute their superiority over the likewise presented linearized formulation.
The aim of finding a method suitable to improve the quantitative prediction of the wave resistance from potential flow is completely fulfilled. Especially the requirement of a procedure that works independent from the pressure integration over the hull, and therefor is independent from undue discretization impact, is met by the transversal wave cut analysis.

From the second test case, which was especially chosen for its notably low resistance, the application of a suitable form factor in the usual application of the resistance determination from towing tank test seems an immediate necessity. Furthermore it appears to the authors that a constant form factor as Prohaska’s method proposes is not always adequate. Fortunately Prohaska’s proposition allows the later determination of a form factor as it only requires the measured data from the tank and the computed values from the ITTC57 correlation line. It can even be used for the determination of linear or higher order form factor. The results for the Wigley test case will also be reviewed under the application of a form factor, which can be obtained from data provided by [11], but which could not be conducted anymore during the preparation of this paper.

Notwithstanding the very pleasing results from the two presented test cases the authors find it absolutely necessary to continue the validation with further test cases of both flavors: experimental test cases and real world applications. Specially the latter should include cases during the early design phase but also comparisons after towing tank tests are performed. Additionally a further engagement concerning the form factor determination is necessary. At the moment the authors are engaged in the preparation of supplementary viscous computations to support their presented findings.

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REFERENCES


[8] Will, J., 2014. MINK is currently under development and has not yet been published.


